Sums of two S-units via Frey-Hellegouarch curves
(joint work with Michael A. Bennett)

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- B. M. M. de Weger (around 1990) : algorithm ; complete list of solutions for $S = \{2, 3, 5, 7\}$.