Processing Heartbeat by fractal analysis

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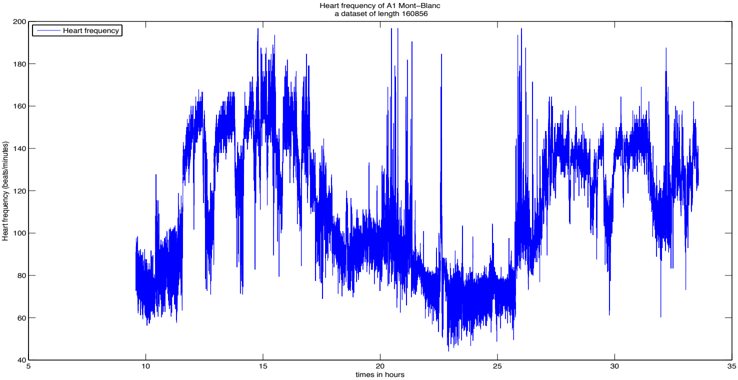
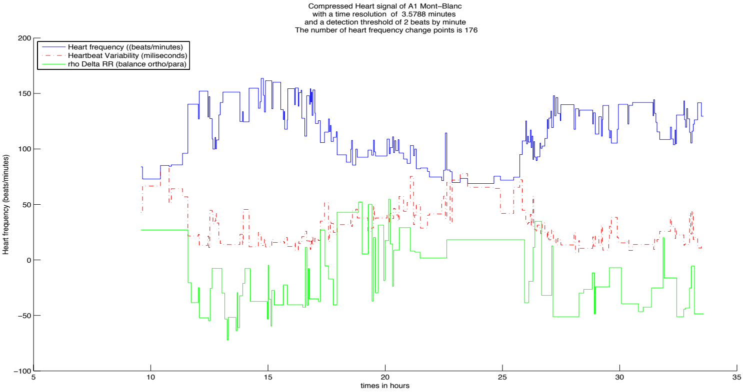
Réunion *"Do Well."*
Saint-Nectaire, 25 juin 2013

Ex. 1) Mont-Blanc climber A1.



Heartbeats series of a Mont-Blanc climber.
Data furnished by team UBIAE, Évry Génopole.

Ex. 1) Mont-Blanc climber A1 (continue).



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Some explanations

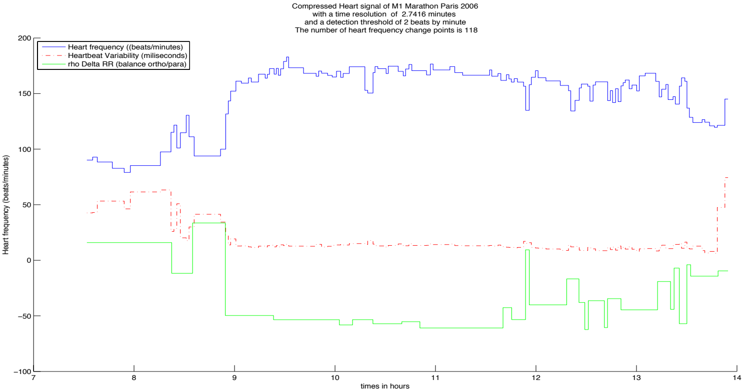
- The blue line is the mean heart frequency ($HR = 60/RR$) after segmentation by FDpV techno.

- The green line is the correlation coefficient ρ of the increments of $X = RR$

$$\rho = corr\Big[X(k+1) - X(k), X(k) - X(k-1)\Big]$$

1. $\rho = 0$ corresponds to a random walk;
2. $\rho > 0$ correspond to persistency;
3. $\rho < 0$ correspond to ant-persistency;

Ex. 2) Marathon runner M1.



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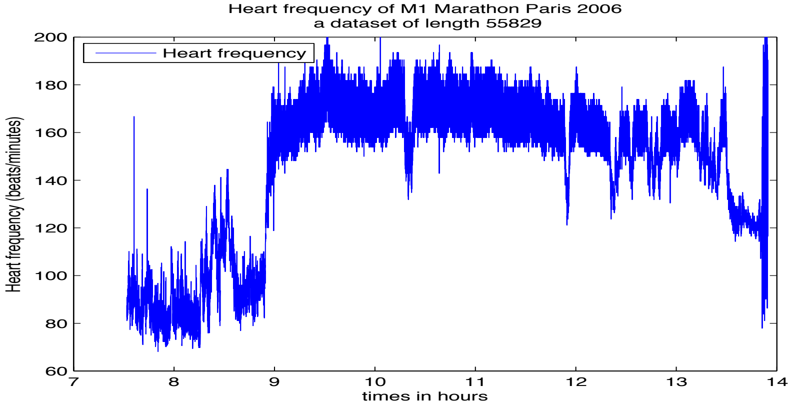

Heartbeats series of a Marathon runner.
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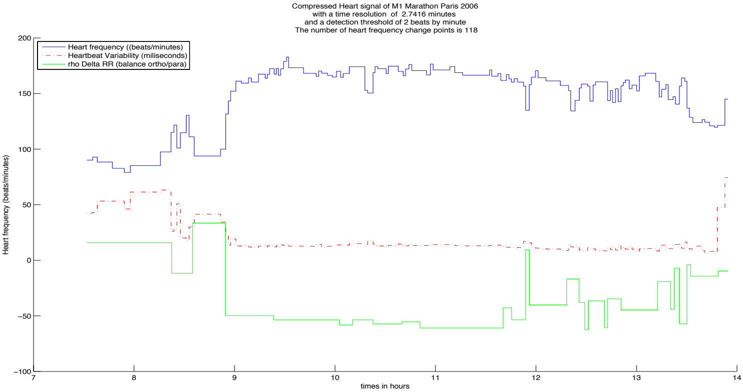
Ex. 2) Marathon runner M1 (continue).



Heartbeats series of a Marathon runner.
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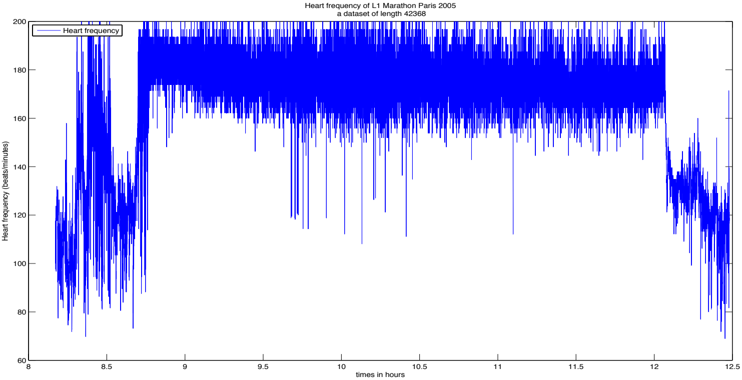
Physiological interpretation

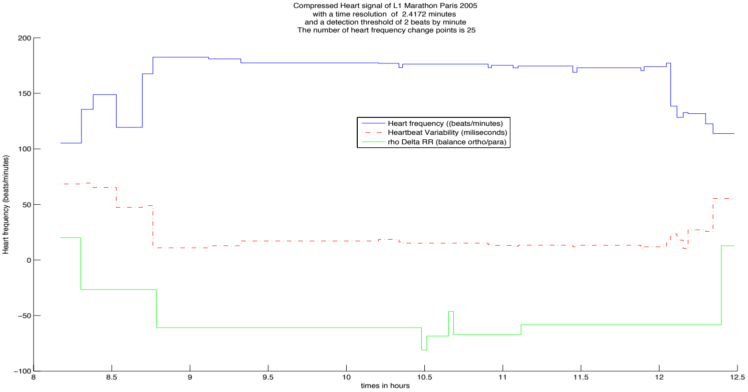
- $\rho \simeq -50/100 < 0$ during the first part of the race (during the 3 first hours).
- $\rho > 0$ at rest before the race.

Ex. 2) Marathon runner L1.



Heartbeats series of a Marathon runner.
Data furnished by team UBIAE, Évry Génopole.

Ex. 2) Marathon runner L1 (continue).



Heartbeats series of a Marathon runner.
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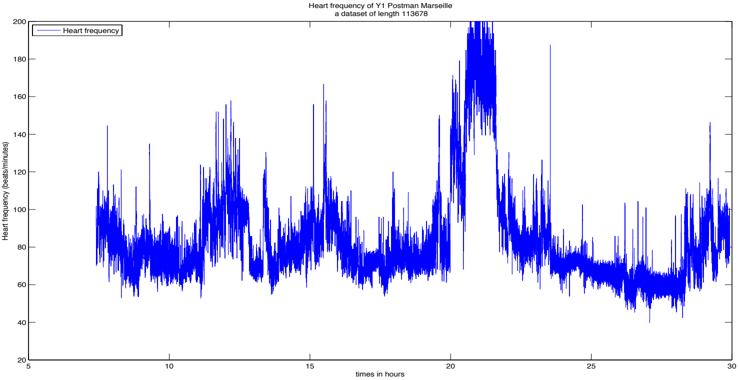
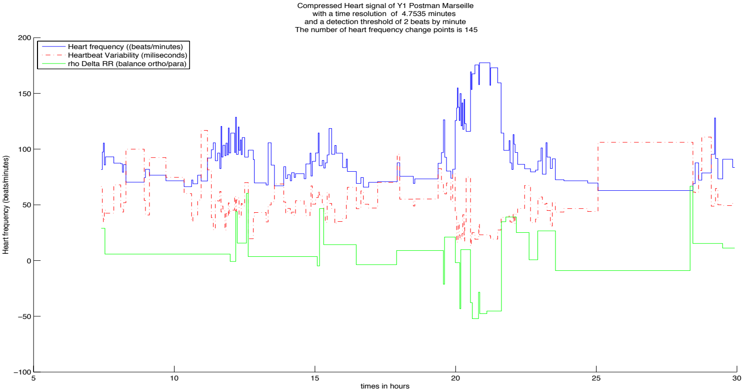
Ex. 3) Shift worker Y1.



Heartbeats series of a shift worker.
Data furnished by Gil Boudet

Ex. 3) Shift worker Y1 (continue).



Heartbeats series of a shift worker.
Data furnished by Gil Boudet

Model

There exists a segmentation $\tau = (\tau_1, \ldots, \tau_K)$ and a family of Hurst indices H $= (H_0, H_1, \ldots, H_K)$ such that

- **X_t is continuous process**
- **and X_t is a fBm with Hurst index H_k on the interval (τ_k, τ_{k+1}) for $k = 0, \ldots, K$**

A possible probabilistic model is furnished by the step fBm (Benassi et al. 2000).

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Change detection on the Hurst index

Our roadmap is to have a fast enough method for estimating the Hurst index, then to plug it into FDpV technology.
We use the increment ratio statistics (IRS), see Bardet and Surgailis, 2011.

Recall on Increment Ratio Statistics (IRS)

Let X be observed at the discrete times $t = 1, \ldots, n$.

1. We define the increments of order $L = 1$ by
$$\Delta_1(t) = X_{t+1} - X_t$$
2. We define the increments of order $L = 2$ by
$$\Delta_2(t) = X_{t+2} - 2X_{t+1} + X_t$$

Then, the IRS is given by

$$IRS_{L,n}(X) = \frac{1}{(n-L)} \sum_{t=1}^{n-L-1} \psi\left(\Delta_L X_t, \Delta_L X_{t+1}\right)$$

with

$$\psi(x,y) := \begin{cases} \dfrac{|x+y|}{|x|+|y|} & \text{if } (x,y) \in \mathbb{R}^2 \backslash \{(0,0)\} \\ 1 & \text{if } (x,y) = (0,0). \end{cases}$$

Central Limit Theorem (CLT) for IRS

When X is a fBm with Hurst index H, that is $X = B_H$, we have the CLT (Bardet, Surgailis, 2011):

$$\sqrt{n}\left(IRS_{L,n}(B_H) - \Lambda_0\left(\rho_L(H)\right)\right) \to \mathcal{N}(0, \Sigma_L^2(H))$$

for $H \in (0,1)$ if $L = 2$ and $H \in (0, 3/4)$ for $L = 1$.
With the asymptotic variance $\Sigma_L^2(H)$ is given by some formula,

$$\Lambda_0(r) \quad := \quad \frac{1}{\pi}\arccos(-r) + \frac{1}{\pi}\sqrt{\frac{1+r}{1-r}}\log\left(\frac{2}{1+r}\right)$$

and

$$\rho_L(H) \quad = \quad \begin{cases} 2^{2H-1} - 1 & \text{if } L = 1 \\ \dfrac{-3^{2H} + 2^{2H+2} - 7}{8 - 2^{2H+1}} & \text{if } L = 2 \end{cases}$$

FDpV for IRS

1. The maps $H \mapsto \rho_L(H)$ and $\rho \mapsto \Lambda_0(\rho)$ are non decreasing.
2. Moreover, when $X = B_H$, we have

$$\mathbb{E}\Big[\psi\left(\Delta_L X_t, \Delta_L X_{t+1}\right)\Big] = \Lambda_0\left(\rho_L(H)\right)$$

- So change on Hurst index for a step fBm, are equivalent to change on the mean of the random variable $\psi\left(\Delta_L X_t, \Delta_L X_{t+1}\right)$.
- IRS can be fast calculate.
- We can apply FDpV method to IRS.

FDpV for IRS

We can apply FDpV method to IRS, with

$$A \times FD_H(t, A) = \sum_{j=t+1}^{t+A} \psi\left(\Delta_L X_j, \Delta_L X_{j+1}\right) - \sum_{j=t-A+1}^{t} \psi\left(\Delta_L X_j, \Delta_L X_{j+1}\right)$$

THANK FOR YOUR ATTENTION

I thank my co-authors :

- Fractal like Processes and Change point analysis:
  A. Benassi- S. Cohen- J.Istas, A. Ayache, J-M. Bardet, M. Fhima, A. Guillin, J. Lévy-Vehel.
- Health and Physiology :
  V. Billat, Gil Boudet, A. Chamoux, F. Dutheil, N. Khalfa.

I thank

- INRIA Saclay where I spent three years (2008-2011)
- Digitéo for the grant "PHYSIOSTAT" (2009-2011)
- The ANR for the grant "DO WELL B." (2013-2016)