Analyse fractale, analyse temps/fréquence

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FDpV for other models

The FDpV method is general, and can be adapted to more complex models:

1. the Toy Model: Sequence of independent r.v.
   - Changes on the mean (BGF, 2011);
   - Changes on the variance (BGF, 2011).
2. Linear Regression
   - Changes on the slope (BGF, 2011);
   - Changes on the intercept (BGF, 2011).
3. Fractal like processes
   - Changes on the Hurst index;
   - Changes on the variance.

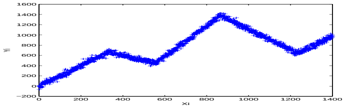
Detection of change points in the slope

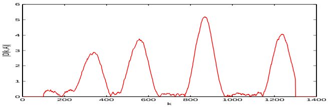
Data

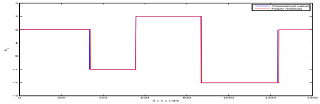
1. $n = 1400$
2. $\Delta = 1, \sigma = 30$
3. $\nu_k \in [3, 5]$ where
   $\nu_k := |a_k - a_{k+1}|$.

Calibration of the FDp-V method

1. $p_1^* = 0.05,$
2. $p_2^* = 10^{-5}$
3. $A = 100$







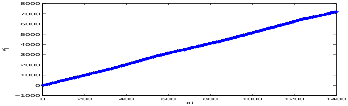
Smaller change points

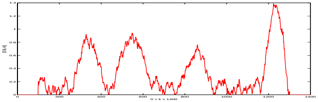
Data

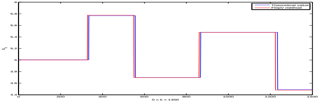
1. $n = 1400$
2. $\Delta = 1$, $\sigma = 30$
3. $\nu_k \in [0.75, 1]$.

Calibration of the FDp-V method

1. $p_1^* = 0.05$
2. $p_2^* = 10^{-5}$
3. $A = 100$







3) FDpV for dynamical models

In this section, we propose more elaborated model for biological series than the toy model of sequence of independent Gaussian r.v. with piecewise constant mean and variance.

We will investigate the two following dynamical models:

1. Locally stationary Gaussian process;
2. Fractal process being piecewise fBm
   (work in progress, see PhD thesis Mehdi Fhima).

The spice of life:

- *" Heartbeats, hormones and health: is variability the spice of life?"*

- Goldberger (2001)
  American J. Critical Care Medicine 163, 1289–1290.

- Ivanov C., Amaral L., Goldberger A.L., Havlin S., Rosenblum M.G., Struzik Z. R., Stanley H. E.(1999). Multifractality in human heartbeat dynamics. *Nature* **399**.

Recall on stationary Gaussian process

Such a process has the following spectral representation (Cramér):

$$X(t) = \mu + \int_R e^{it\xi} f^{1/2}(\xi)\, dW(\xi), \quad \text{for all} \quad t \in R, \qquad (1)$$

where

- dW is a complex Wiener measure chosen such that X is real value;
- the power spectral density $f(\xi)$ is a non negative even function. Moreover Formula (1) is well defined under the condition
$$\int_R f(\xi)\, d\xi < \infty.$$

Fractional Brownian motion and Gaussian process with stationary increments

- Fractional Brownian motion (fBm) has been introduced by Kolmogorov (1940) and popularized for its relevance in application by Mandlebrot & Van Ness (1968).
- FBm is a particular case of zero mean stationary Gaussian process with spectral representation:

$$X(t) = \int_R (e^{it\xi} - 1) \cdot f^{1/2}(\xi)\, dW(\xi), \quad \text{for all} \quad t \in R, \quad (2)$$

corresponding to a linear spectral density:

$$\log\left(f(\xi)\right) = -(2H + 1)\log(|\xi|) + \sigma^2$$

where H is the Hurst index and σ a scale parameter (variance).

FBm and Gaussian process

- The spectral density of fBm is an affine function of frequency (log-log), and this corresponds to the self-similarity of fBM.

Following Mandelbrot, an important feature of f.B.m. is "*the concept of self-similarity, a form of invariance with respect to changes of time scale.*"

- Formula (2) is well defined under the condition

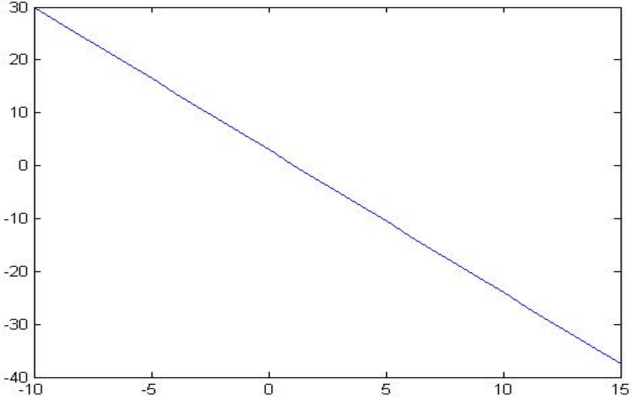
$$\int_R \left|e^{it\xi} - 1\right|^2 \cdot f(\xi)\, d\xi < \infty.$$

- Self-similarity implies affinity (log-log scale) of spectral density, then

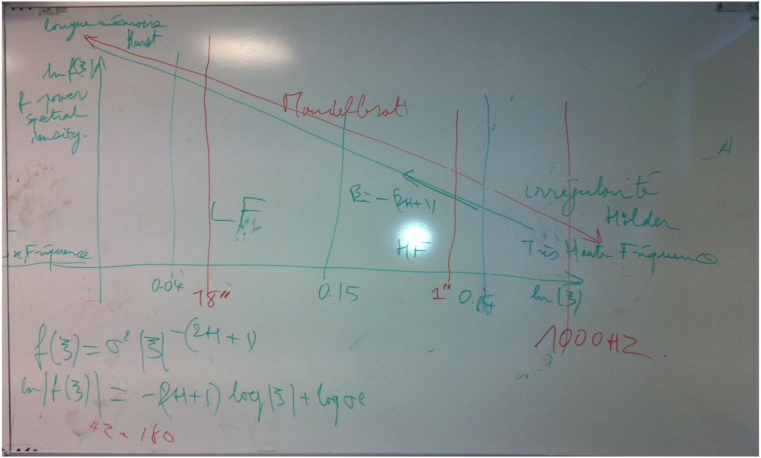
$$\int_R \sigma^2 \left|e^{it\xi} - 1\right|^2 \cdot |\xi|^{-(2H+1)}\, d\xi < \infty,$$

and after $H \in (0,1)$.

Spectral density of FBm



Spectral density of Multiscale FBm, to be continue



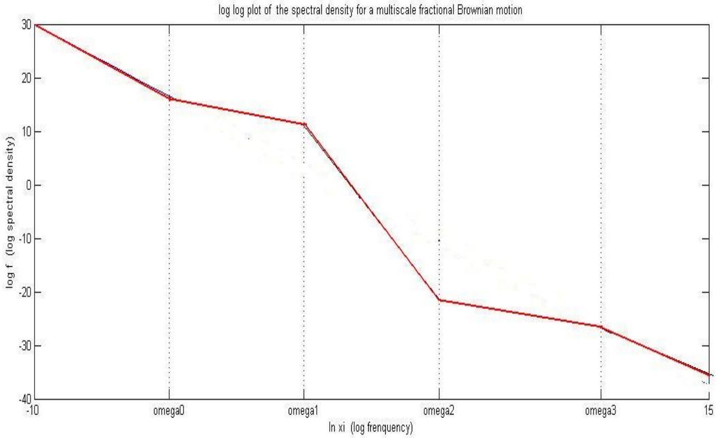
Multiscale FBm

- FBm is a particular case of Gaussian process with stationary increments depending on two parameters: $H \in (0, 1)$ and $\sigma > 0$.

- In some applications (see Bardet & Bertrand 2007), we find Hurst index $H \notin (0, 1)$.

This paradox is solved by the multiscale fBm, which is still a zero mean stationary Gaussian process, but with a piecewise affine spectral density (log-log scale) (Bardet & Bertrand, 2007b)

Spectral density of Multiscale FBm



Cardiologist point of view

Cardiologists are interested in the study of heartbeats series in two frequency bands:

1. The hight frequency (HF) bands (0.15 Hz, 0.5 Hz) assumed to reflect regulation by the orthosympathetic system.
2. The low frequency (LF) bands (0.04 Hz, 0.15 Hz) assumed to reflect regulation by the parasympathetic system.

Task force of the European Society of Cardiology and the North American Society of Pacing and Electrophysiology (1996).

ECG and Tachogram

Autonomous nervous system (ANS)

Noradrénaline & other agents

(Ortho)Sympathetic system

Acetylcholine & other agents

Parasympathetic system

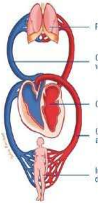
For the cardiac activity

Accelerate heart activity

Decelerate heart activity

High frequency band of the HRV (0.15Hz,0.5Hz)

Low frequency band of the HRV (0.04Hz,0.15Hz)



- Cardiac activity is an interesting indicator of the ANS state
- Interested fields: Cardiology , physiology, psychology (well being)
- Analysis :Time domain, frequency domain and non linear methods

A new Approch for Analysing Heart Rate Regulation, Khalfa et al. (2011)

Statistical estimation of spectral density

In Bardet & Bertrand (2010), we have made an estimation of the spectral density of heartbeat series for a shift worker (C1) by wavelet analysis.

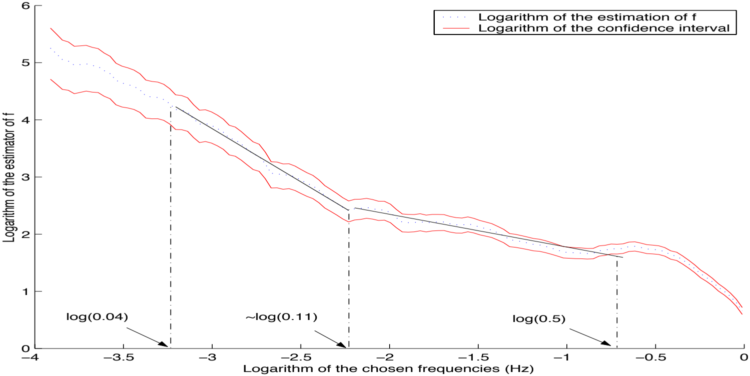


Figure: spectral density (log-log plot) of C1, at work

Statistical estimation of spectral density following the activity

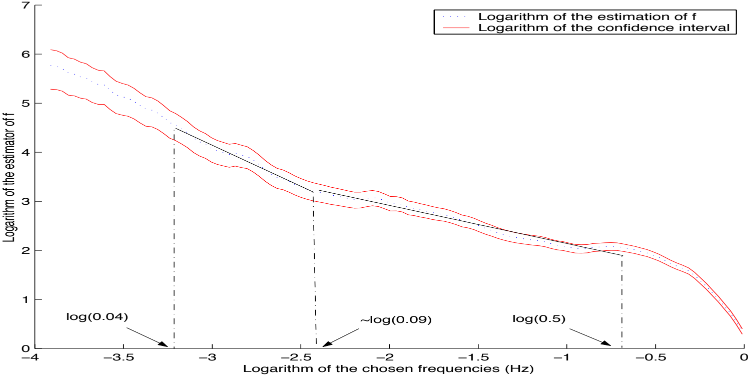


Figure: spectral density (log-log plot) of C1, at rest

Statistical estimation of spectral density following the activity

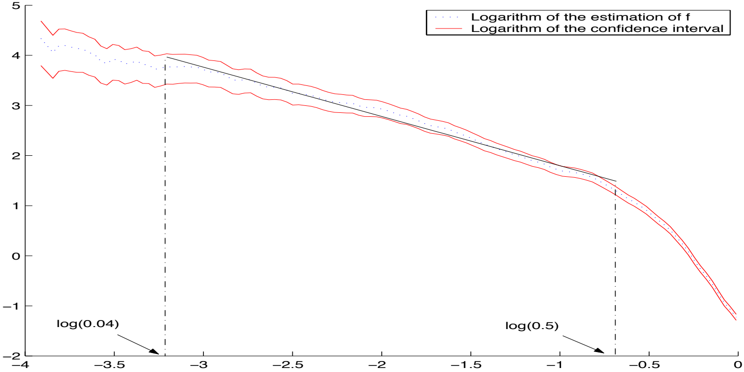


Figure: spectral density (log-log plot) of C1, sleeping

Locally stationary Gaussian process

Such a process has the following spectral representation (Cramér):

$$X(t) = \mu(t) + \int_R e^{it\xi} f^{1/2}(t,\xi)\, dW(\xi), \quad \text{for all} \quad t \in R, \quad (3)$$

where

- dW is a complex Wiener measure chosen such that X is real value;
- the spectral density $f(t,\xi)$ is a piecewise constant function of time, i.e., there exists a partition $\tau_1, \ldots, \tau_K$ such that $f(t,\xi) = f_k(\xi)$ for $t \in [\tau_k, \tau_{k+1}[$;
- the mean $\mu(t)$ is also piecewise constant for another partition $\widetilde{\tau}_1, \ldots, \widetilde{\tau}_L$ with $\mu(t) = \mu_\ell$ if $t \in [\widetilde{\tau}_\ell, \widetilde{\tau}_{\ell+1})$.

Statistical analysis of orthosympathetic and parasympathetic frequency bands

1. We calculate the wavelet coefficient in HF and LF bands,

$$W_{\psi_j}(b) = \int_R \psi_j(t-b)\,X(t)\,dt \qquad \text{for } j = 1 \text{ or } 2. \quad (4)$$

   where the wavelet ψ_j has a frequency support well localized in the HF or LF frequency band, with time support L_j
2. We calculate the corresponding log-wavelet energies.
3. We use FDpV method to segment log-wavelet energies.

Statistical analysis of HF and LF frequency bands (Mathematical formulas)

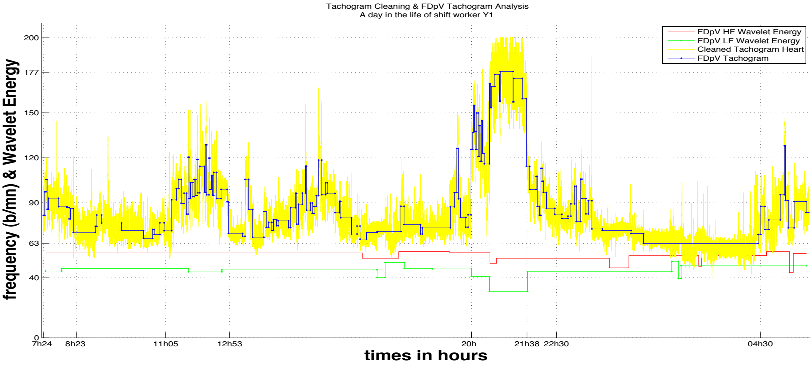
- The harmonizable representation of wavelet coefficients (Bardet & PRB 2010 or Ayache & PRB 2011)

$$W_j(b) = \int_R e^{ib\xi}\,\widehat{\psi}_j(\xi)\,f_k^{1/2}(\xi)\,dW(\xi)$$

  for all $(b, b+L_j) \subset (\tau_k, \tau_{k+1})$.
- Thus $W_j(b)$ is a zero mean complex valued Gaussian process with variance $E\left(|W_j(b)|^2\right) = \int_R \left|\widehat{\psi}_j(\xi)\right|^2 f_k(\xi)\,d\xi$ where $|z|$ denotes the modulus of the complex number z.

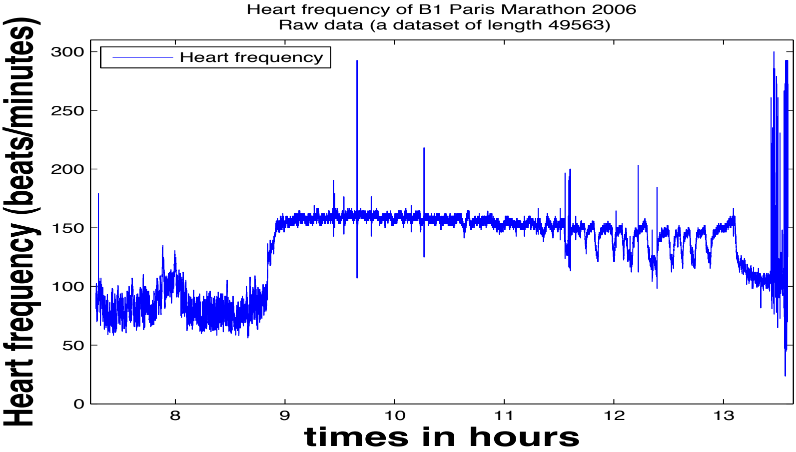
In Vivo Tachogram Analysis (In ViTA)

By using software "*InVi*TA", we get for shift worker Y1



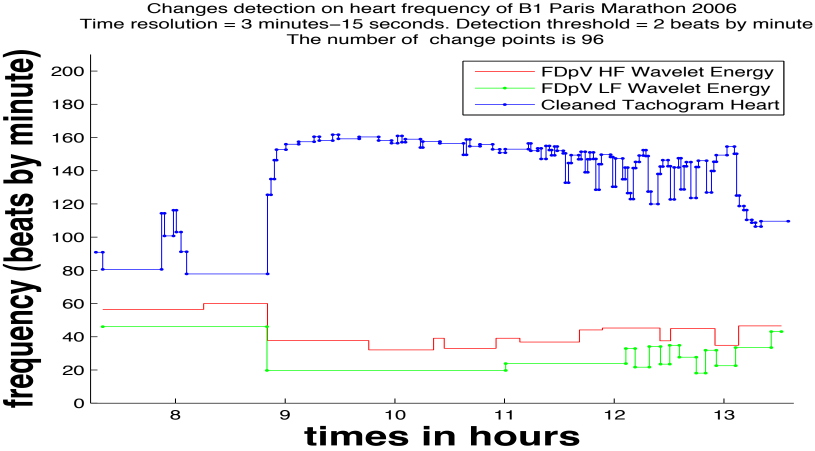
4) An index of well-being

Heartbeat series for a Marathon runner (B1)



Data furnished by V.Billat and LEPhE (laboratory of exercise physiology, INSERM and Évry Genopole).

In Vivo Tachogram Analysis (In ViTA) for Marathon runner B1



Conclusion

New data, new models and new statistical methods allow new applications to be developed:

- New devices can record high frequency physiological signals inside and outside the laboratory. These signals are digital, indexed by time, fluctuating and with big data size.
- Generalizations of the fractional Brownian motion with Hurst index varying have been introduced since 1996, and several statistical estimators have been proposed.
- Change point analysis can accurately describe time varying parameters. However big datasets also require to control the numerical efficiency of statistical procedures.

References

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